"An Evaluation of the Handling Qualities of Seven General-Aviation Aircraft," TN D-3726, Nov. 1966, NASA.

[6]Sachs, G., "Untersuchung über die Auswirkungen von Zusatzfedern und Zusatzgewichten auf die dynamische Längsstabilität," IFD Bericht 2/73, 1973, Institut für Flugtechnik der Technischen Hochschule Darmstadt, Darmstadt, Germany.

Potential Flow Past Annular Aerofoils

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Nomenclature

F_p = prescribed normal velocity boundary condition
G = strength of mass flow control vortex
K = strength of Kutta vortex
N = number of panels used to approximate an annular aerofoil surface
$\vec{n}$ = unit normal vector to a surface or panel
P = a general field point in space or a point on the boundary surface
q = a source point on the body surface
Q = surface source density
V = velocity
V_{nij} = normal component of the induced velocity at the control point of ith surface panel by a unit value of source density of the jth surface panel
V_{tij} = tangential component of the induced velocity at the control point of the ith surface panel by a unit value of source density on the jth surface panel.
θ = surface slope of the aerofoil

Subscripts

i = Denotes quantities associated with the ith panel
j = Denotes quantities associated with the jth panel
p = Denotes quantities associated with the boundary
∞ = Denotes quantities associated with the free-stream

THE problem of the subsonic potential flow past annular aerofoils has been treated extensively, recently using the method of singularities.[1,6] Smith and Hess[1] solved the problem by using surface sources and computing the flow properties from the resulting integral equation using high-speed digital computers. Geissler[3] has recently solved the thick annular aerofoil problem using a combination of source and vortex rings. An interesting experimental technique based on the linear theory using rheoelectric analogy[4] has been used for both the direct and inverse problems.

The purpose of this Note is to study the lowspeed potential flow past annular aerofoils using three different methods. The Kutta condition is satisfied at the trailing edge and the mass flow through the body is specified independently. The first is the Young's nonlinear method[2] using a combination of source and vortex distributions. The second is the rheoelectric analogy in which the flow potential is simulated in an inclined tank using the electrical analogy. The third is the Smith's method wherein source distributions are put on the body. All these techniques are applied to NACA-66-006 profile and the relative merits of the three assessed.

Analysis

The basic integral equation for a continuous source distribution on any arbitrary body is given by

$$2\pi Q(P) - \int_S\int \frac{\partial}{\partial n}\left(\frac{1}{r(p,q)}\right) Q(q)dS = -\vec{n}\cdot\vec{V}_\infty + F_p \tag{1}$$

As explained in Ref. 1, the annular aerofoil is approximated by a large number of panels in the form of conical frustra. The continuous distribution of sources over the body surfaces is replaced by N discrete values, each constant over a segment. The mid point of each segment is chosen as the control point. A surface source strength of unit strength is placed on each element and the normal velocity component to the body surface induced at every control point by all other elements is calculated by numerical integration.

The mass flow through the annular aerofoil can be changed by the addition of a uniform vortex distribution placed on the camber surface of the aerofoil and extending downstream of the trailing edge on the mean cylinder. The strength of this vortex can be varied to give the required intake flow and this induces a normal velocity component at the control points on the surface of the ring aerofoil.

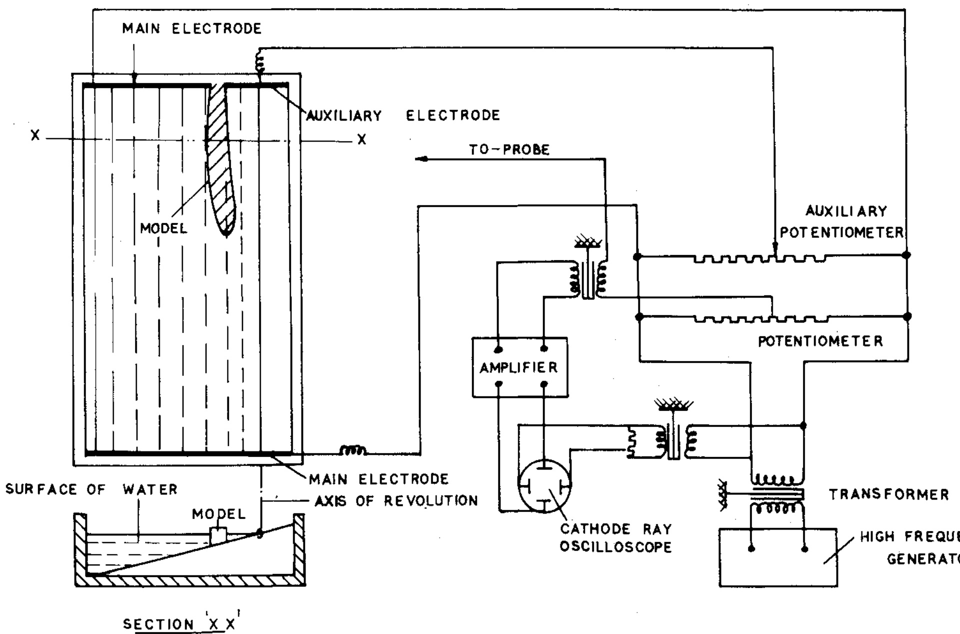

Fig. 1 Rheoelectric analogy-inclined tank, simulation of flow past annular aerofoils with different inlet flow conditions.

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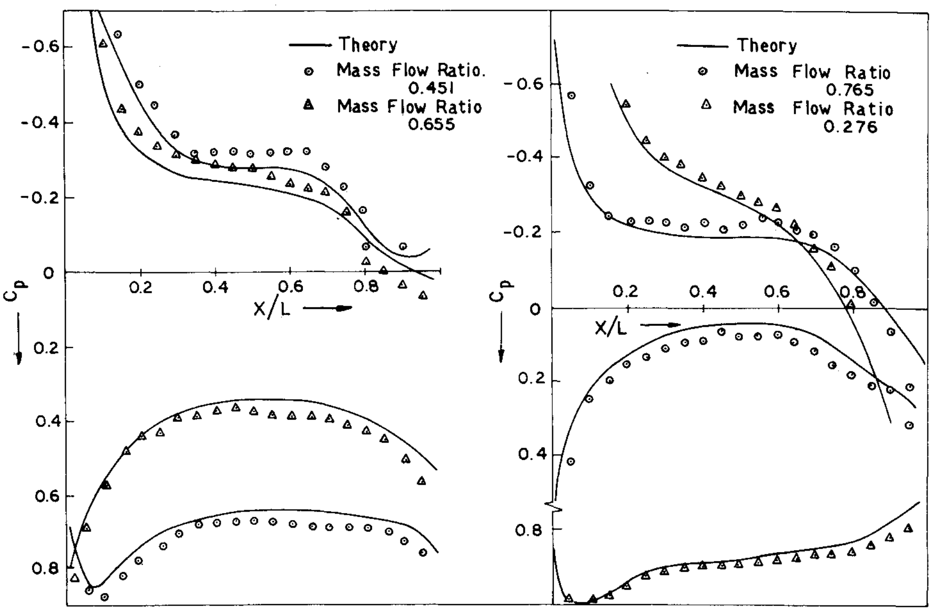

Fig. 2 Comparison of nonlinear theory and rheoelectric analogy for NACA 66-066, profile fineness ratio 0.8: a) without hemisphere central body; and b) with hemisphere cylinder central body.

The circulation around the aerofoil is undefined until Kutta condition is applied at the trailing edge. Another uniform vortex distribution is placed on the camber surface and only extends over the chord length. This allows for the velocity jump across the trailing edge due to the trailing vortex cylinder. This kutta vortex distribution induces a normal velocity at the control points. Now applying the boundary condition that the normal velocity is zero, at each control point, the following equations are obtained.

$$\sum_{j=1}^{N-1} V_{nij}Q_j + V_{ni}{}'K = V_\infty \sin\theta_i - V_{ni}{}'G, \quad i = 1,2,\ldots(N-1) \qquad (2)$$

$$\sum_{j=1}^{N-1} (V_{t1,j} + V_{t_{N-1},j})Q_j - (V_{t_1} + V_{t_{N-1}})K = V_\infty(\cos\theta_1 + \cos\theta_{N-1}) + (V_{t_1}{}' + V_{t_{N-1}})G + G, \quad i = 1,2,\ldots(N-1) \qquad (3)$$

The singularity strengths can be calculated as explained in Ref. 2 and thereby the tangential velocities and the pressure coefficients can be computed at all the control points.

The axisymmetric potential flow past an annular aerofoil is simulated in an inclined tank of dimension 150 cm × 75 cm × 30 cm fabricated out of wood. The tank along with the accessories are shown in Fig. 1. The feed electrodes are made of 10 mm. Aluminium sheet. In this work, the input supply of 15v and 400 cycles/sec is used. The potentiometers used in the bridge circuits can measure the potentials up to 1/1000th of the applied voltage. The model consists of a meridian section formed of the NACA aerofoil, made of wood. The electrical potential distribution measured over the contour of the model is converted into velocity potential as explained in Ref. 4 and the velocity and pressure distributions are computed numerically.

Results and Discussion

The pressure distribution obtained by Young's nonlinear method as well as the rheoelectric analogy as applied to NACA 66-006 for two different mass flow ratios are given in Fig. 2. Figure 2b gives the pressure distribution on the annular aerofoil with a hemispherical nosed cylindrical body kept with nose at the leading edge. Figure 3 gives the results of Smith's theory without controlling mass flow ratio as applied to same annular aerofoil and the results compared with the available wind tunnel tests.[5]

Although in Ref. 1, Smith describes the procedure for controlling the mass flow ratio for annular aerofoils wherein the original configuration is altered to give a semi-infi-

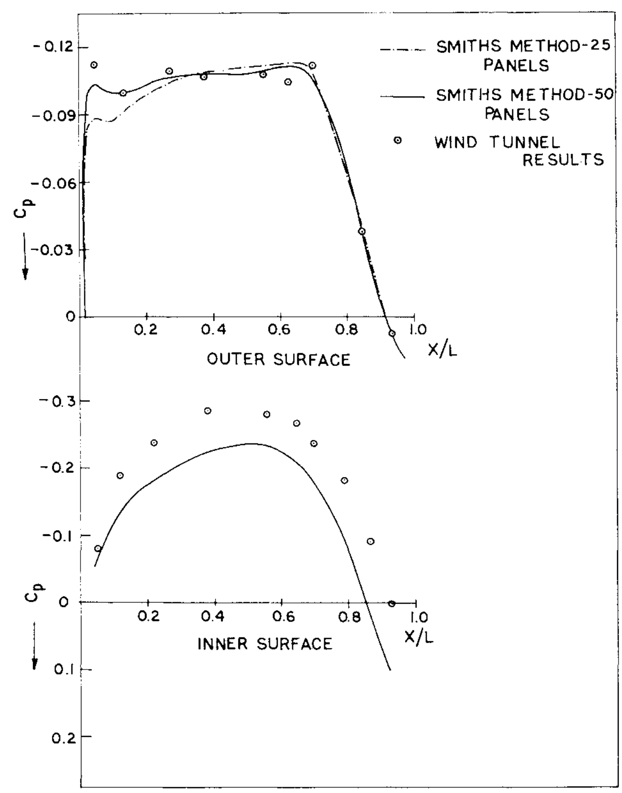

Fig. 3 Comparison of Smith's method and wind-tunnel results for NACA 66-066 aerofoil.

nite after body it is not clear how this could be applied to finite annular bodies. The numerical experimentation reveals that the distribution of panels should be such that the variation of source strengths should be gradual without any abrupt jumps. As mass flow ratio increases, the source strength decreases for the internal flow and increases for the external flow. Regarding panel distribution, sixty panels are used, with 30 panels on internal surface and 30 panels on external surface with one point at the leading edge. In the case of annular aerofoil with the central body, about 51 points are distributed on the aerofoil and 10 points on the central body. Even though the number of panels is very much less than what Smith has suggested,[1] the comparison is fairly good. The agreement near the trailing edge is poor. The reason may be due to the interference effects of the electrodes kept near the trailing edge for controlling the mass flow.

Conclusion

From the above discussion it is clear that the rheoelectric analogy is a good first approximation in solving the flow problems past annular aerofoils with and without central bodies. The main advantage in its use is the simplicity, and the continuous representation of the flowfield without involving any discretisation. Comparison with Young's nonlinear theory is fairly good. Smith's method using source distributions gives a good representation of flow past thick semi-infinite annular aerofoils without controlling mass flow, as shown in Fig. 3.

References

[1]Hess, J. L. and Smith, A. M. O., "Calculation of Potential Flow about Arbitrary Bodies," *Progress in Aeronautical Sciences,* Vol. 8, Pergamon Press, New York, 1966.

[2]Young, C., "A Computer Program to Calculate the Pressure Distribution on an Annular Aerofoil," ARC-CP 1217, 1972, Aeronautical.

[3]Geissler, W., "Berechrung der Potentialstromung um Rotationsymmetrische Ring Profile," AVA-Bericht 71A02, Gottingen, 1971.

[4]Malavard, L. and Hacques, G., "Problems de L'aile Annulaire Resolns par Analogie Rheoelectrique," *Advances in Aeronautical Sciences,* Vol. 12, Pergamon Press, New York, 1959.

[5]Hough, G. R., "The Aerodynamic Loading on Streamlined Duct Bodies," *Development in Mechanics,* Vol. 2, Pt. 1, Pergamon Press, New York, 1965.

[6]Belotserkovskii, S. M., "The Theory of Thin Wings in Subsonic Flow," Plenum Press, New York, 1967.

Roll-Up of Aircraft Trailing Vortices using Artificial Viscosity

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Nomenclature

b = span
c = chord
C_L = lift coefficient
r = radial coordinate
U = freestream velocity
u = axial velocity component
v = spanwise velocity component
w = vertical velocity component
x = streamwise coordinate
y = spanwise coordinate
z = vertical coordinate
Γ = circulation
κ = trailing vortex strength
ν = kinematic viscosity
$\bar{\nu}$ = $\nu/(Uc)$

Subscripts

$(\ )_c$ = vortex center
$(\ )_f$ = flap

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Introduction

IN the past several years considerable interest has been shown in computational methods of analyzing the roll-up characteristics of aircraft trailing vortices. Although the general problem is quite complicated, considerable progress has been made by assuming a potential flow model. This idea was first introduced by Westwater.[1] Since that time, various applications and extensions of the Westwater type of method have been made.[2-5] Examination of the computational results of these investigations reveals that all of these authors encountered an irregular roll-up of vortex sheet in the tip region.

In a recent study of the numerical representation of vortex sheets Kuwahara and Takami[6] found that by including an "artificial viscous core" term in the equation for induced velocity with a suitably chosen value of the "artificial viscosity coefficient" the irregularities that normally occur in the Westwater method can be smoothed out. (A similar approach also has been proposed by Chorin and Bernard.[7])

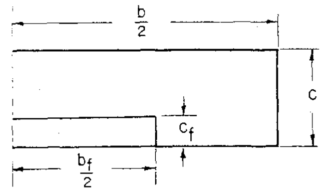
The purpose of the present investigation was to apply the artificial viscosity method of Kuwahara and Takami to a number of practical aerodynamic configurations and compare the results for the core location with existing experimental data where possible.

Mathematical Model

Following Kuwahara and Takami the induced velocity field of a collection of two dimensional line vortices with artificial viscosity distributed over the span of the wing is given by

$$u_i = \frac{dx_i}{dt} = U \text{ or } x_i = t_i$$

$$v_i = \frac{dy_i}{dt} = -\frac{1}{2\pi}\sum_{j\neq i}\kappa_i\left[1-\exp\left(\frac{-r_{ij}^2}{4\nu t}\right)\right]\frac{(z_i - z_j)}{r_{ij}^2}$$



Configuration Number	Description	C_L	c_f/c	δ_f	b_f/b	b	c
1	Plain Wing	1.0	-	-	-	8	1
2	Flapped Wing	1.5	0.25	20	0.5	8	1
3	Flapped Wing	1.0	0.25	40	0.5	8	1

Fig. 1 Wing-flap geometry and aerodynamic configurations.